Numerical insight into flow structure in ultraturbulent thermal convection

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Very large eddy simulations of high-aspect-ratio unbounded Rayleigh-Bénard convection for Pr=0.71 over a 10-decade range of Rayleigh numbers ($Ra=10^5-10^{15}$) reveal a consolidation and dramatic thinning of the wall boundary layer with an increase in the Ra number. The fingerlike plumes between planform structures become also thinner, more distant, but much more vigorous. The Ra exponent in the $Nu\propto Ra^n$ correlation follows $n\approx0.31$ scaling, but n begins to increase gradually above $Ra=10^{13}$. However, no trend towards "crossing" of the thermal and hydrodynamic boundary layers is observed.

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Much controversy surrounds the physics of turbulent Rayleigh-Bénard (RB) convection, especially at high Rayleigh (Ra) numbers. The focus is on the $Nu\propto Ra^n$ correlation (where Nu is Nusselt number), but the scaling controversy reflects the general disagreement about the underlying physics and heat transfer mechanism. The interest is motivated not only by scientific curiosity, but also by the importance of the RB phenomenon in understanding thermal convection in atmosphere, oceans, earth's mantle, and in many technological applications. The classic exponent $n=1/3$ implies that heat flux is governed by processes confined within the wall boundary layers and that plumes generated on one wall never reach the other, i.e., there is no direct communication between the walls. While this may sound reasonable at low Ra numbers, it seems unlikely at high Ra since persistent plumes (that may break into thermals or puffs) stretching from one wall to another wall have been observed both experimentally and in numerical simulations [1–4]. Wall communication involves a different heat transfer scenario, resulting in a different Ra exponent, e.g. Siggia [5]. Most evidence seems to support 2/7 scaling ("hard" regime) observed by Castaing *et al.* [6] for $Ra>4\times10^7$ in their experiment with helium. Other experiments using water report the 1/3 power law for $Ra>10^9$ with a smaller exponent for low Ra numbers [5b]. While the small difference in the Ra exponent may not be a reliable indicator, reports on other parameters, which should provide better indication of the change in the regime, are also inconsistent.

Even larger controversy surrounds the very high Ra numbers, roughly beyond 10^{11} or higher, for which Chavanne and co-workers [7–9] observed another "ultrahard" regime characterized by an increase in the Ra exponent to about 0.38–0.39 (for $10^{11}<Ra<2\times10^{14}$), indicating a trend toward the asymptotic value of 1/2 for infinite Ra numbers. They reported other anomalies in the ultimate regime such as a dramatic change in the spatial organization of the temperature fluctuations and transition from the supposedly laminar to turbulent wall boundary layers. Recently, these finding have been disputed by Glazier *et al.* [10] and Niemela *et al.* [11] based on their experiments with mercury and helium, respectively. The disagreement between different findings could arise from differences in the experimental setups, Pr number, cell size, and geometry (rectangular-circular), difficulties in controlling the experiment (heat leakage), and from different width/height aspect ratios. Although the latter is claimed to have no significant effect on scaling if larger than 1.0 (e.g., Chavanne *et al.* [8]), it is difficult to accept that the effects of sidewalls and corners, even if the thermal conditions are perfectly controlled, will not be influential through wall friction and blocking effects. Experimental evidence of convective rolls sweeping across the entire domain, confirmed by our simulations, with horizontal wavelengths expanding with an increase in the Ra number if there is no sidewall blocking, is just one of the indications of the sidewall effects. Observations at a single location, usually along the vertical line in the cell center, may not be sufficient for drawing conclusions about the flow structure in the entire domain.

Numerical simulations offer an alternative route to solving turbulent thermal convection. Direct numerical simulation (DNS), which resolves all scales up to the Kolmogorov ones, is limited to low and moderate Ra numbers. A simple analysis based on the ratio between the dissipation length scale and the distance between the walls, Kerr [1], $\eta/H=1/H(\nu^3/\varepsilon)^{1/4}=[Pr^2/(Nu-1)Ra]^{1/4}$ and using Niemela *et al.* [11] correlation, $Nu=0.124\ Ra^{0.309}$, shows that the fully resolved simulations require 135, 33 000, and 250 000 grid points in the vertical direction for $Ra=10^7$, 2×10^{14}, 10^{17}, respectively. With the corresponding limit on the time step and the need to perform the time integration over at least a few convective time scales, this puts, at present, the upper limit on the achievable Ra by DNS to $\approx10^{10}$, even when using massively parallelized computers. Large-eddy simulations (LES), which fully resolve larger scales of turbulent motion while modeling the scales smaller than the characteristic grid size, sound as a viable alternative for high Re and Ra flows. Yet, handling complex geometries and resolving the flow very close to walls, are challenges that still pertain. The third classical approach is the Reynolds averaged Navier-Stokes (RANS) method. Here all fluctuations are time averaged, so that no information about eddy scale and its dynamics in spectral space can be retrieved.

In order to overcome problems associated with each numerical approach, we applied a transient RANS (TRANS) method. Here we resolve in time and space the large deterministic eddy structure (very large eddy simulations), while the rest of turbulence, covering still a substantial portion of the stochastic turbulence spectrum, is modeled using a one-point algebraic closure model [12,13]. While the TRANS of

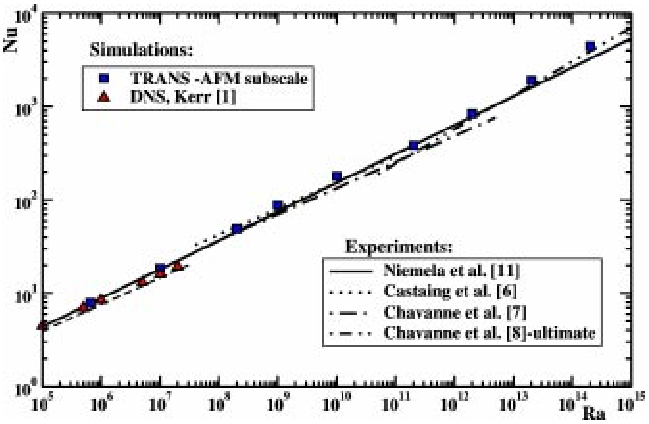

FIG. 1. Comparison of the computed Nu(Ra) results with several experimental and DNS correlations over a range of Ra numbers.

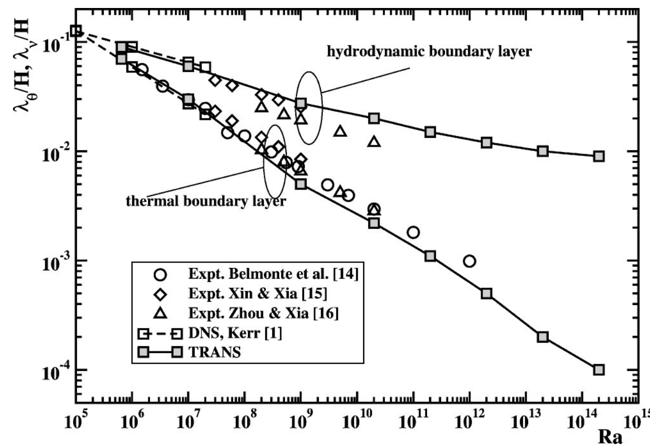

FIG. 2. Thickness of thermal (λ_θ/H) and hydrodynamical (λ_v/H) boundary layers as functions of Ra.

a specific, confined, low-aspect-ratio cell poses no problem, we considered an unbounded large-aspect (8:8:1) ratio RB convection that removes at least one of the uncertainties: the effect of the sidewalls. The simulations covered 10–20 convective time scales for each Ra in the range $10^9 \leqslant \mathrm{Ra} \leqslant 2\times10^{14}$. Each new case was started from the interpolated solutions for the previous consecutive Ra number. This reduced drastically the total computation costs: for the entire series of eight Ra numbers the total computation time was approximately the same as that reported by Kerr [1] for $\mathrm{Ra}=2\times10^7$ for just one convective time scale.

Comparison of the computed integral Nu results with several experimental and DNS correlations is shown in Fig. 1. The TRANS simulations show good agreement with the available DNS and experimental correlations over the entire range of Ra numbers. Agreement is particularly good for low and intermediate Ra numbers where the difference between the TRANS, DNS, and experimental correlation of Niemela *et al.* [11] is less than 2%. For the last two highest simulated Ra's, i.e., $\mathrm{Ra}=2\times10^{13}$ and 2×10^{14}, the numerical results show an increase in Nu by 15% and 25%, respectively, as compared with the experimental results of Niemela *et al.* [11], while following closely the correlations of Chavanne and co-workers $\mathrm{Nu}=105(\mathrm{Ra}/10^{10})^{0.365}$, [7,8], with difference being less than 10%. Admittedly, the results for only two highest Ra considered may not be fully representative, especially in view of possible effects of Pr number, which in the experiment deviated significantly for these Ra numbers, whereas in the simulations it was kept constant.

The dependence of the nondimensional thickness of the hydrodynamic (velocity) and thermal boundary layers, λ_v/H and λ_θ/H, on Ra is shown in Fig. 2. λ_v and λ_θ were determined from the positions where the total kinetic energy $(\overline{\langle k\rangle}+0.5\widetilde{U}_i\widetilde{U}_i)$ and temperature variance $(\overline{\langle\theta^2\rangle}+\widetilde{T}\widetilde{T})$, respectively, reach their maximum. In the low Ra range ($\mathrm{Ra}\leqslant2\times10^7$), our simulations are in excellent agreement with the DNS of Kerr [1] and Kerr and Herring [4] for both thicknesses. In this range, λ_θ/H scales best with $\mathrm{Ra}^{-1/3}$, and λ_v/H with $\mathrm{Ra}^{-1/7}$. In the intermediate range of Ra, $2\times10^7\leqslant\mathrm{Ra}\leqslant10^{10}$, we have compared λ_θ and λ_v with different sets of experimental data, i.e., Belmonte *et al.* [14], Xin and Xia [15], and Zhou and Xia [16]. It is recalled that different experimental studies show quite different behaviors despite all being performed in the same range of Ra numbers, $10^6\leqslant\mathrm{Ra}\leqslant10^{11}$. Experimental results of Belmonte *et al.* [14] demonstrated that λ_v/H is larger than λ_θ/H and that the gap between them is *decreasing* as Ra increases. They estimated that a potential crossover between the boundary layers should occur beyond $\mathrm{Ra}\approx10^{14}$ (the highest Ra reported in their experiment was 10^{11}). Contrary to Belmonte *et al.* [14] results, the experiments of both Xin and Xia [15] and Zhou and Xia [16] demonstrated that λ_v and λ_θ will not cross at higher Ra. The reason for this disagreement was associated with difficulties in estimating experimentally λ_v. Belmonte *et al.* [14] defined λ_θ as a distance at which the linearly extrapolated near-wall mean temperature profile equals the reference temperature. In the experiment with water, they observed that the position of the maximum mean horizontal velocity coincides with the position of the maximum cutoff frequency in the temperature power spectrum. They then applied this similarity to measure *indirectly* the location of the maximum velocity and its dependence on Ra for gas as working fluid. Xin and Xia [15] *directly* measured both the mean and fluctuating velocity profiles using a light scattering technique. More recently, Zhou and Xia [16] have determined the viscous boundary layer thickness by measuring spatial correlation of the temperature fluctuations between two thermistor probes located perpendicular to the main flow. They showed that the obtained results were in excellent agreement with the direct measurements, thereby giving confidence in the proposed method. Our numerical results follow exactly the measured trends in λ_θ for all three experiments, Fig. 2. It seems that λ_θ/H scales very well with $\mathrm{Ra}^{-1/3}$ over the entire simulated range of Ra numbers, $10^5\leqslant\mathrm{Ra}\leqslant10^{15}$, whereas λ_v/H follows perfectly the experimental data of Xin and Xia [15] in $10^7\leqslant\mathrm{Ra}\leqslant10^9$ range. Just above $\mathrm{Ra}=10^{12}$ our simulations indicate a change in slope. Note that this change corresponds to the position where an increase in the slope of the Nu−Ra correlation occurs, Fig. 1. This enhancement in heat transfer together with the observed change in the slope of λ_v/H gives an indication of a possible new regime observed in Ref. [9].

The existence of large-scale organized structures at very high Ra numbers has also been disputed. Several experiments reported coherent large-scale circulation that persists to the highest Ra number attained in laboratory (10^{14}) [5]. We argue that convective rolls indeed exist, created by rising and falling thermal plumes, irrespective of Ra number, though with an increase in Ra they become more and more vigorous, erratic, and elongated in the horizontal direction. The rolls create shear at the edge of the wall boundary layer, though another mechanism of the creation of the boundary layers is also probable: the plume release sites create local pressure lows which attract surrounding fluid generating a well established and self-maintaining horizontal wall boundary layer. The mechanism tends to maintain itself resulting in unstable but continuous plumes that at high Ra numbers break into thermals or puffs. The plumes are the main carriers of heat as shown by maximum heat transfer coefficients at plume impingement and minimum at their release sites [12]. In order to get a better insight into the flow and heat transfer mechanism we provide some visualization of the large-scale structures. The visualized fields show a network of polygonal cells (planform structures) with fingerlike plumes in between. These plumes move randomly and interact with each other causing a strong vertical motion (dark regions at the peak of temperature isosurface). A very similar pattern of planform structures was observed in experiments of Theerthan and Arakeri [17], giving a further confidence in the here proposed numerical approach. In essence, the thermal plumes originating from boundary layers rise and create strong vertical updrafts. They can penetrate up to the opposite wall when they start to be deflected, creating a strong horizontal motion that directly affects the boundary layer thickness and controls in return the thermal plume bursts. We consider these interactions as one sequence of a single phenomenom, i.e., it is impossible to analyze separately the behavior of thermal plumes and large coherent convective structures because of strong coupling between the velocity and thermal field. The structure and dynamic interaction between thermal plumes and large-scale convective rolls significantly change as Ra increases. For large Ra numbers planform structures are significantly thinner due to stronger horizontal movement in the boundary layers as a result of more intensive mixing which ought to transfer higher heat rates. As a consequence, a larger distance between thermal plumes is observed compared to lower Ra numbers, Fig. 3.

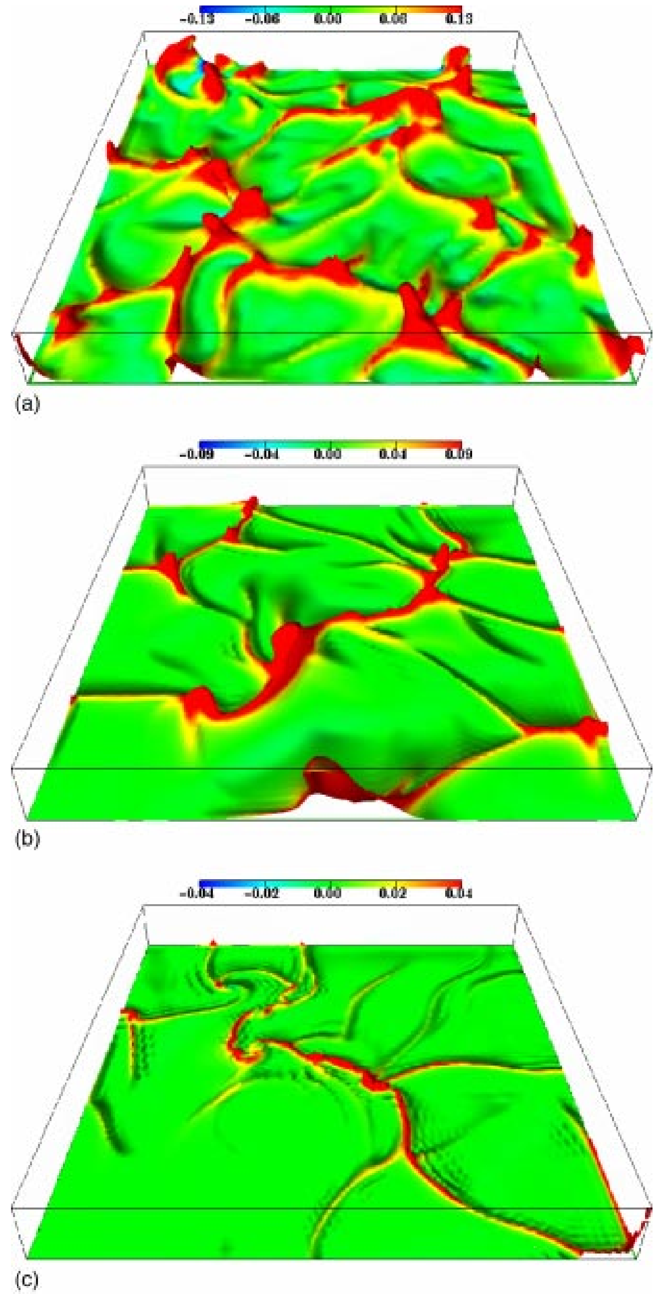

FIG. 3. Planform structures with fingerlike plumes in between (temperature isosurfaces colored with intensity of vertical velocity) for $\mathrm{Ra}=6.5\times10^5$, 10^9, 2×10^{14}.

In order to portray changes in the flow structures as Ra increases, we show in Fig. 4 the horizontal projection of the instantaneous trajectories of 64^2 massless particles in the near-wall (inside the hydrodynamic boundary layer) and central ($z/H=0.5$) region. Despite complex flow morphology, several distinct features can be observed: regions with strong one-dimensional movements (dark lines), divergent stagnation regions (unstable focal points—where the dark lines merge), and regions with strong and very well defined plane circulation (roll structure). The first two flow regimes can be best observed in a plane inside the boundary layer (left column) and the circular roll pattern in the central plane (right column), Fig. 4. Note the change in flow morphology as Ra increases. For low (6.5×10^5) and moderate (10^9) Ra numbers in the central plane the roll structures are stronger and more dense than for very high Ra number (2×10^{14}). This corresponds to the observed wavelength change of the planform structures discussed earlier. Significant differences can also be observed in the near-wall region. The horizontal movement in the boundary layers becomes more intensive as Ra increases. For the very high Ra, strong roll structures are located deeply inside of the hydrodynamic boundary layer.

To summarize, by performing TRANS-based VLES, we

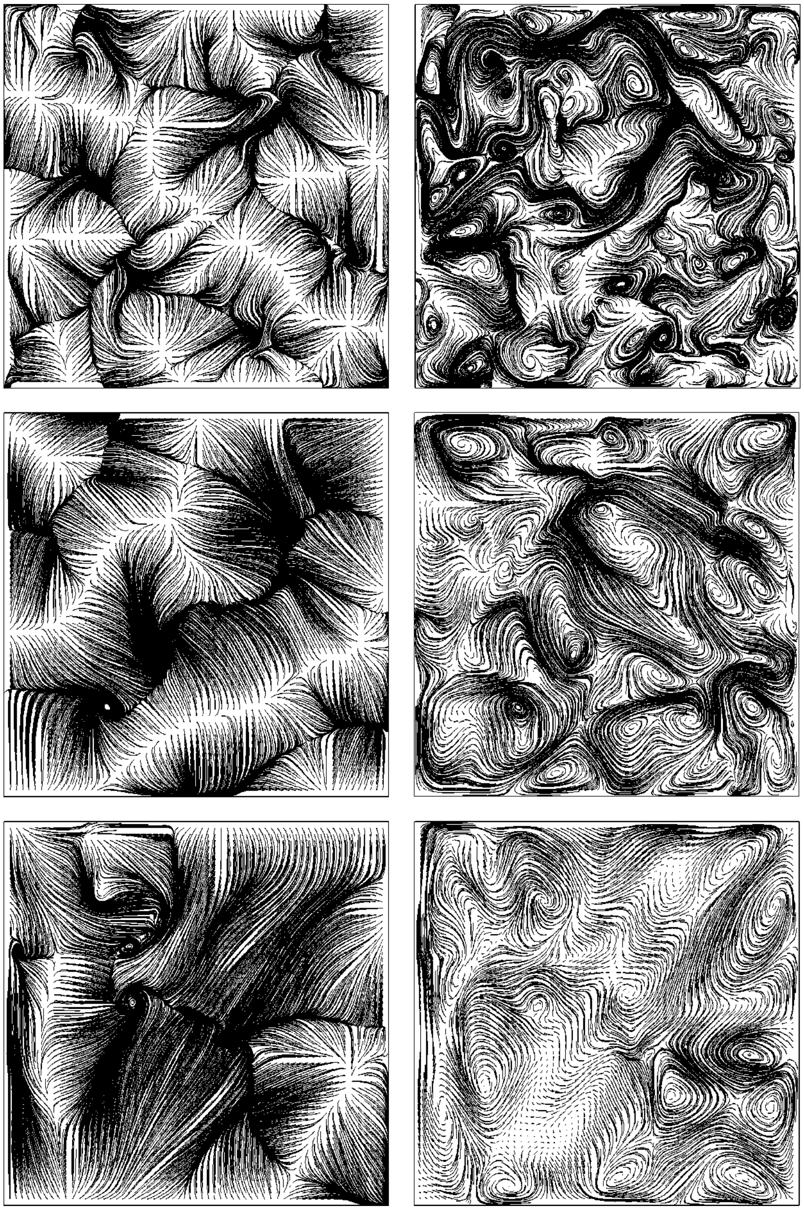

FIG. 4. Instantaneous trajectories in the central horizontal plane (right) and inside the hydrodynamical boundary layer (left) for Ra$=6.5\times10^5$, 10^9, 2×10^{14}.

provide numerical solutions to RB convection with walls at constant temperatures for a broad range of Ra numbers that includes also the ultraturbulent regime ($10^5<\mathrm{Ra}<10^{15}$). Such high Ra numbers are inaccessible to LES or DNS and are also very difficult for experiments. This 10-decade range of Ra provided a basis for a comparison of different scaling proposals and enabled testing numerically the Nu-Ra controversy and to gain insight into roll and cell structures at very high Ra numbers. The simulated results are in very good agreement with DNS (for low Ra) and with experiments, including the recent experimental results of Niemela *et al.* [11]. For the last two highest values of Ra (2×10^{13},2×10^{14}) a change in the slope of hydrodynamic boundary layer thickness and a reorganization of planform and plume structures were observed together with an enhancement in heat transfer, indicating a possibility of a regime experimentally observed by Chavanne and co-workers [7–9].

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